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THE EFFECT OF THE REINFORCEMENT STRUCTURE ON THE HEAT CONDUCTIVITY OF SHELLS OF REVOLUTION WITH A SYSTEM OF TUBES FILLED WITH A LIQUID HEAT-TRANSFER AGENT

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The initial boundary-value heat-conduction problem for shells reinforced by tubes filled with a flowing liquid heat-transfer agent is considered. The dependence of the coefficients in the heat-conduction equations on the thermophysical characteristics of the composition phases, reinforcement parameters, and shell geometry is studied. A comparative analysis of the stationary temperature fields in thin shells of revolution of different Gaussian curvature is performed for various reinforcement structures and heat-exchange regimes. It is shown that the temperature distribution in the shells depends strongly on the reinforcement structure and the shell geometry, which opens up new possibilities of designing optimal structures.

The structural elements designed for heat accumulation and transfer are widely used in modern power installations, transport systems, jet engines in aerospace engineering, laser and MHD installations, etc. The potentialities of using homogeneous materials in these installations have almost been exhausted. Further progress is associated with composite structures, which ensure the discrete, continuous, or discrete-continuous distribution of thermophysical characteristics and heat sources. These structures can be reinforced by curvilinear tubes filled with a flowing liquid heat-transfer agent. The heat conductivity of these structures equipped with a system of "heat tubes" has not yet been investigated.

1. Formulation of the Problem. We consider a shell reinforced by N families of constant-cross-section tubes filled with an incompressible fluid. In the absence of internal heat sources and in the orthogonal curvilinear coordinates x_i ($i = 1, 2, 3$), the linear heat conduction of the shell is governed by the following system [1]:

$$CT_{,t} = (A_1A_2A_3)^{-1}\sum_{i=1}^{3}\Big(A_1A_2A_3A_i^{-1}\sum_{j=1}^{3}aA_j^{-1}\Lambda_{ij}T_{,j}\Big)_{,i}$$

$$+\sum_k\Big[\lambda_k(A_1A_2A_3)^{-1}\sum_{i=1}^{3}(A_1A_2A_3A_i^{-2}\omega_kT_{k,i})_{,i} - \lambda_k\omega_k\partial_k^2(T_k) - 2h_kr_k^{-1}\omega_k(T-T_k)\Big]; \qquad (1.1)$$

$$c_k\rho_kT_{k,t} + c_k\rho_kv_k\partial_k(T_k) = \lambda_k\partial_k^2(T_k) + 2h_kr_k^{-1}(T-T_k), \quad k = 1, 2, \ldots, N; \qquad (1.2)$$

$$\Lambda_{ij} = \sum_k\Omega_k\Omega^{-1}\{l_{ki}l_{kj}a^{-1}[(\lambda_{1k}-\lambda_c)\Omega + a\lambda_c] + (-1)^{i+j}l_{kp}l_{kr}a[\Omega\lambda_{2k}^{-1} + (a-\Omega)\lambda_c^{-1}]^{-1}\},$$

$$p = 3-i, \qquad r = 3-j, \qquad i, j = 1, 2, \qquad (1.3)$$

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$$\Lambda_{33} = a\left[\sum_k \Omega_k \lambda_{2k}^{-1} + (a-\Omega)\lambda_c^{-1}\right]^{-1}, \quad C = c_c\rho_c A + \sum_k \Omega_k C_k R_k;$$

$$\Omega = \sum_k \Omega_k, \qquad a = 1 - \sum_k \omega_k, \qquad A = a - \Omega; \tag{1.4}$$

$$\partial_k = l_{k1} A_1^{-1} \frac{\partial}{\partial x_1} + l_{k2} A_2^{-1} \frac{\partial}{\partial x_2}, \quad l_{k1} = \cos\alpha_k, \quad l_{k2} = \sin\alpha_k. \tag{1.5}$$

Here T is the temperature of the composite shell, C and Λ_{ij} are the reduced heat capacity and effective heat conductivities of the shell, respectively, λ_k, c_k, ρ_k, and T_k are the linear heat conductivity, specific heat, bulk density, and temperature of the fluid that fills tubes of the kth family, respectively, ρ_c and R_k are the bulk densities of the binding material and the material of the tubes of the kth family, respectively, c_c and C_k are the specific heats of the binding material and the tubes of the kth family, respectively, λ_c, λ_{1k}, and λ_{2k} are the linear heat conductivities of the isotropic binder and the transversely isotropic tube of the kth family in the longitudinal and transverse directions, respectively, $r_k = \text{const}$ is the internal cross-sectional radius of the tubes of the kth family, is the coefficient of heat exchange between the tubes of the kth family and the fluid, v_k is the longitudinal component of the fluid flow rate in the tubes of the kth family [since the fluid is incompressible ($\rho_k = \text{const}$) and the cross sections of the tubes are constant, we have $v_k = \text{const}$ along the tube axis], Ω_k and ω_k are the intensity of reinforcement by the tubes of the kth family and the intensity of the fluid that fills these tubes (volume content of the fluid of the kth family in the volume element of the heat exchanger), α_k is the angle of reinforcement by the tubes of the kth family, reckoned from the x_1 direction, and A_i are the Lamé parameters; the fixed value of the parameter x_3 corresponds to the elementary reinforced layer, and summation is performed from 1 to N if the limits are not indicated. The quantities ω_k, Ω_k, and A must satisfy the inequalities

$$\omega_k \geqslant 0, \quad \Omega_k \geqslant 0 \quad (k = 1, 2, \ldots, N), \quad A = 1 - \sum_k (\omega_k + \Omega_k) > 0. \tag{1.6}$$

The conditions of constant cross sections and constant internal cross sections of the tubes of the kth family have the form [2]

$$(A_2 A_3 \omega_k l_{k1})_{,1} + (A_1 A_3 \omega_k l_{k2})_{,2} = 0, \quad (A_2 A_3 \Omega_k l_{k1})_{,1} + (A_1 A_3 \Omega_k l_{k2})_{,2} = 0 \quad (1 \leqslant k \leqslant N). \tag{1.7}$$

Consequently, the complete system of equations that describes the linear heat conductivity of a shell reinforced by heat tubes of constant cross section is determined by equations and relations (1.1)–(1.7), which must be supplemented by the initial and boundary conditions for the temperatures T and T_k. At the edge S_k^ω where the tubes of the kth family enter the shell, the values of the functions ω_k and Ω_k must be specified [2].

It is noteworthy that, in practice, the heat conductivities, specific heat, and intensity of the internal heat sources of a composite material are assumed to be known from experiments. However, analysis of relations (1.1), (1.3), and (1.4) shows that in heat tube-reinforced shells, the reduced heat conductivities, the specific heat, and the efficiency of heat exchange depend greatly not only on the thermophysical characteristics of the phases of the composition, but also on the reinforcement parameters: the angle α_k, the intensities Ω_k and ω_k, and the dimension of the tube internal cross section r_k.

2. Heat Conductivity of Thin Shells of Revolution Reinforced by Heat Tubes. In analysis of shell-type thin-walled structures, it is expedient to reduce the three-dimensional heat-conduction problem described by system (1.1), (1.2) to a two-dimensional problem. To this end, we employ the Bubnov–Galerkin procedure in the variable x_3, assuming that the coefficients of corresponding expansions of the functions T and T_k depend on the variables t, x_1, and x_2. Moreover, in view of the fact that the shell is thin, we can confine our analysis to three terms in these expansions [3]:

$$T = \sum_{n=0}^{2} T^{(n)}(t, x_1, x_2)(x_3)^n, \quad T_k = \sum_{n=0}^{2} T_k^{(n)}(t, x_1, x_2)(x_3)^n \quad (k = 1, 2, \ldots, N). \tag{2.1}$$

The Lamé parameters A_i ($A_3 = 1$), the reinforcement parameters Ω_k, ω_k, and α_k, and the functions C and Λ_{ij} can be assumed to be independent of x_3 [2].

We assume that the heat exchange between the ambient medium and the faces of the shell of thickness $H = 2h =$ const obeys the Newton law. As a result, the closed system of equations that describes the heat conduction of this structure has the following form:

$$C\Theta_{,t} = (A_1A_2)^{-1}\sum_{i=1,2}\Big(A_1A_2A_i^{-1}\sum_{j=1,2} aA_j^{-1}\Lambda_{ij}\Theta_{,j}\Big)_{,i}$$

$$+\sum_k\Big[\lambda_k(A_1A_2)^{-1}\sum_{i=1,2}(A_1A_2A_i^{-2}\omega_k\Theta_{k,i})_{,i} - \lambda_k\omega_k\partial_k^2(\Theta_k)$$

$$- 2h_kr_k^{-1}\omega_k(\Theta - \Theta_k) + 15\lambda_k\omega_k(\theta_k - \Theta_k/H)/h\Big] + B\Theta + D_-T_{-\infty} + D_+T_{+\infty},$$

$$c_k\rho_k\Theta_{k,t} + c_k\rho_kv_k\partial_k(\Theta_k) = \lambda_k\partial_k^2(\Theta_k) + 2h_kr_k^{-1}(\Theta - \Theta_k),$$

$$c_k\rho_kT^{(1)}_{k,t} + c_k\rho_kv_k\partial_k(T_k^{(1)})$$

$$= \lambda_k\partial_k^2(T_k^{(1)}) + 2h_kr_k^{-1}\{[(a_{22} - a_{12})\Theta/H - a_{22}T_{+\infty} + a_{12}T_{-\infty}]/\Delta - T_k^{(1)}\}, \quad (2.2)$$

$$c_k\rho_k\theta_{k,t} + c_k\rho_kv_k\partial_k(\theta_k) = \lambda_k\partial_k^2(\theta_k) + 2h_kr_k^{-1}\{\Theta/H + 4h^2[(a_{11} - a_{21})\Theta/H$$

$$+ a_{21}T_{+\infty} - a_{11}T_{-\infty}]/(15\Delta) - \theta_k\},$$

$$T^{(0)} = \Theta/H - h^2T^{(1)}/3, \quad T^{(1)} = [(a_{22} - a_{12})\Theta/H - a_{22}T_{+\infty} + a_{12}T_{-\infty}]/\Delta,$$

$$T^{(2)} = [(a_{11} - a_{21})\Theta/H + a_{21}T_{+\infty} - a_{11}T_{-\infty}]/\Delta, \quad T_k^{(0)} = (9\Theta_k/H - 5\theta_k)/4,$$

$$T_k^{(2)} = 15(\theta_k - \Theta_k/H)/H^2 \qquad (k = 1, 2, \ldots, N),$$

Here $B = a\{-(\mu_+ + \mu_-)[H^{-1} + h(a_{11} - a_{21})]/(3\Delta) + (\mu_- - \mu_+)(a_{22} - a_{12})/(2\Delta)\}$, $D_+ = a[\mu_+ + (\mu_+ - \mu_-)ha_{22}/\Delta - (\mu_+ + \mu_-)Hha_{21}/(3\Delta)]$, $D_- = a[\mu_- + (\mu_- - \mu_+)ha_{12}/\Delta + (\mu_+ + \mu_-)Hha_{11}/(3\Delta)]$, $\Delta = a_{11}a_{22} - a_{12}a_{21}$, $a_{11} = -(m_+ + h)$, $a_{12} = -H(m_+ + h/3)$, $a_{21} = m_- + h$, $a_{22} = -H(m_- + h/3)$, $m_\pm = \Lambda_{33}/\mu_\pm$, and $T_{\pm\infty}$ and $\mu_\pm$ are, respectively, the ambient temperature and the coefficients of heat exchange between the shell and the ambient medium from the side of the "external" (subscript plus) and "internal" (subscript minus) faces. Similar equations can be obtained in the case where the different temperature and heat-flux boundary conditions or mixed boundary conditions [1] are specified on the shell faces.

Consequently, for thin shells, the three-dimensional heat-conduction equations (1.1) and (1.2) are reduced with sufficient accuracy to equations of the type (2.2) for the desired functions Θ, Θ_k, $T_k^{(1)}$, and θ_k, which depend only on the time t and two spatial variables x_1 and x_2.

To formulate the initial boundary-value problem corresponding to system (2.2), we integrate the initial and boundary conditions over the thickness of the shell. As a result, we obtain the initial and boundary conditions for the functions Θ, Θ_k, $T_k^{(1)}$, and θ_k.

We use the example of simple structures to illustrate the effect of the reinforcement structure on the temperature field. Below, we confine ourselves to thin shells of revolution reinforced axisymmetrically over equidistant surfaces.

Since the initial boundary-value problem corresponding to system (2.2) is linear and its solution is periodic in the x_2 coordinate, we can expand the desired functions Θ, Θ_k, $T_k^{(1)}$, and θ_k and the known functions $T_{\pm\infty}$ in Fourier series in x_2 [4] and reduce the problem to a system of ordinary differential equations.

Upon the axisymmetric thermal action, these equations become

$$CT_{,t} = (A_1R)^{-1}(RA_1^{-1}a\Lambda_{11}T_{,1})_{,1} + \sum_k [\lambda_k(A_1R)^{-1}(RA_1^{-1}\omega_kT_{k,1})_{,1}$$

$$- \lambda_k\omega_kl_{k1}A_1^{-1}(l_{k1}A_1^{-1}T_{k,1})_{,1} - 2h_kr_k^{-1}\omega_k(T - T_k)],$$

$$c_k\rho_kT_{k,t} + c_k\rho_kv_kl_{k1}A_1^{-1}T_{k,1} = \lambda_kl_{k1}A_1^{-1}(l_{k1}A_1^{-1}T_{k,1})_{,1} + 2h_kr_k^{-1}(T - T_k), \tag{2.3}$$

$$k = 1, 2, \ldots, N,$$

where $T = T^{(0)} = \Theta/H$ and $T_k = T_k^{(0)} = \Theta_k/H$.

3. Solution of the Problem and Discussion of Results. We consider the heat conduction for three types of shells of revolution characterized by:

1) The zero Gaussian curvature ($K = 0$) (conical shell):

$$R(x_1) = [(x_1 - x_1^0)R^1 - (x_1 - x_1^1)R^0]/(x_1^1 - x_1^0). \tag{3.1}$$

2) The positive Gaussian curvature ($K > 0$) [the shell shaped like an elliptic paraboloid (SSEP)]:

$$R(x_1) = a\sqrt{x_1 - c} + b, \tag{3.2}$$

where

$$a = (R^1 - R^0)\Big(\sqrt{x_1^1 - c} - \sqrt{x_1^0 - c}\Big)^{-1}, \quad b = R^0 - a\sqrt{x_1^0 - c}, \quad c < x_1^0. \tag{3.3}$$

3) The negative Gaussian curvature ($K < 0$) [the shell shaped like a one-sheeted hyperboloid of revolution (SSOH)]:

$$R(x_1) = \sqrt{a^2 + b^2(x_1 - c)^2}, \tag{3.4}$$

where

$$a^2 = [(R^0)^2(x_1^1 - c)^2 - (R^1)^2(x_1^0 - c)^2]/[(x_1^1 - c)^2 - (x_1^0 - c)^2],$$

$$b^2 = [(R^1)^2 - (R^0)^2]/[(x_1^1 - c)^2 - (x_1^0 - c)^2], \qquad c^0 \leqslant c \leqslant c^1, \tag{3.5}$$

$$c^0 = (R^1x_1^0 - R^0x_1^1)/(R^1 - R^0), \qquad c^1 = (R^0x_1^1 + R^1x_1^0)/(R^1 + R^0).$$

In relations (3.1), (3.3), and (3.5), it is assumed that $x_1^1 > x_1^0$, $R^1 > R^0$, and $R^i = R(x_1^i)$, where $i = 0, 1$; the parameter c enables us to specify the families of shells (3.2) and (3.4); note that the SSEP and SSOH degenerate into conical shells as $c \to -\infty$ and $c \to c^0$, c^1, respectively.

In the axisymmetric case, the conditions of constant cross-sectional areas of the tubes (1.7) have the form [2]

$$R\Omega_k \cos\alpha_k = \Omega_{*k} = \text{const}, \quad R\omega_k \cos\alpha_k = \omega_{*k} = \text{const} \quad (k = 1, 2, \ldots, N), \tag{3.6}$$

where Ω_{*k} and ω_{*k} determine the total cross-sectional areas and internal cross-sectional areas of the tubes of the kth family, respectively, with an accuracy to constant multiplier. These parameters can be the initial data of the problem.

We study the effect of the reinforcement structure on the temperature field in shells with equal characteristic dimensions (the lengths along the axes of revolution and the radii of the edges) for the same boundary conditions. To compare different reinforcement variants, we use the criterion of equal total cross-sectional areas Ω_{*k} and equal internal cross-sectional areas ω_{*k} of the tubes of the kth family, which corresponds to the equal fluid flow rate per unit time for any reinforcement variants at the same values of v_k.

We consider the shells of revolution with edges of the radii R^0 and R^1 ($R^1 = 3R^0$) for $x_1^0 = 0$ and $x_1^1 = 3R^0$, respectively. The shells are made of copper [$\lambda_c = 400$ W/(m · deg), $c_c = 419$ J/(kg · deg), and $\rho_c = 8940$ kg/m^3 [5]) and reinforced by two families ($N = 2$) of steel tubes [$\lambda_{ik} = 45$ W/(m · deg), $C_k = 568$ J/(kg · deg), and $R_k = 7780$ kg/m^3, where i, $k = 1, 2$]. The tubes are located symmetrically in the meridional direction ($\alpha_2 = -\alpha_1$) and characterized by the intensities $\omega_1 = \omega_2$, $\Omega_1 = \Omega_2$, $\omega_{*1} = \omega_{*2}$,

$\Omega_{*1} = \Omega_{*2}$, and $\Omega_{*k} = 0.25\omega_{*k}$. The quantities ω_{*k} in (3.6) are determined from the additional condition $A \geqslant 0.2$. The tubes are filled with transformer oil ($\lambda_k = 0.107$ W/(m · deg), $c_k = 1905$ J/(kg · deg), and $\rho_k = 856$ kg/m^3 [6]) which flows with the same velocity ($v_1 = v_2$).

The equations governing the stationary axisymmetric heat conduction of the shells under the conditions of thermal insulation at its faces are obtained from (2.3) by ignoring terms which contain the partial derivatives with respect to t. We write these equations in dimensionless form:

$$(A_1 r)^{-1}(rA_1^{-1}a\bar{\Lambda}_{11}T')' + 2\varepsilon[(A_1 r)^{-1}(rA_1^{-1}\omega_1 T_1')'$$
$$- \omega_1 A_1^{-1}\cos\alpha_1(A_1^{-1}\cos\alpha_1 T_1')'] - 2\omega_1 H_1(T - T_1) = 0; \tag{3.7}$$

$$\varepsilon A_1^{-1}\cos\alpha_1(A_1^{-1}\cos\alpha_1 T_1')' - V_1 A_1^{-1}\cos\alpha_1 T_1' + H_1(T - T_1) = 0, \quad T_2 = T_1, \tag{3.8}$$

where

$$\varepsilon = \lambda_1\lambda_c^{-1}, \qquad V_1 = c_1\rho_1 v_1 R^0\lambda_c^{-1},$$
$$H_1 = 2h_1(R^0)^2/(r_1\lambda_c), \quad \bar{\Lambda}_{11} = \Lambda_{11}\lambda_c^{-1}, \quad r = R/R^0 \tag{3.9}$$

(the prime denotes differentiation with respect to the dimensionless variable $x = x_1/R^0$).

For the above-chosen materials of the binder and tubes, the coefficient $\bar{\Lambda}_{11}$ in (3.7) and (3.9) is of order 1 and $\varepsilon = 2.675 \cdot 10^{-4}$. For $R^0 = 1$ m and $v_1 = 0.01$ m/sec, we obtain $V_1 = 40.77$. The constant H_1 in (3.7)–(3.9) can be of order 1 provided the internal diameter of the tubes $2r_1$ is sufficiently small. Consequently, ε is a small parameter. The authors showed in [1] that by virtue of the small value of ε, it suffices to use the asymptotic solution of system (3.8), (3.9). To this end, the terms containing ε are ignored in the system, which simplifies the corresponding boundary-value problem. In this case, the function T_1 needs only one boundary condition at the edge S_1^ω, where the fluid flows into the shell: $T_1(S_1^\omega) = T_{1S}$.

We analyze the following reinforcement variants: 1) meridional reinforcement where two families of tubes are located in the meridional directions ($\alpha_1 = \alpha_2 = 0$); 2) spiral reinforcement where the tubes are located at an angle $\alpha_k = (-1)^k\pi/4$ ($k = 1, 2$); 3) spiral reinforcement where the tubes are located at an angle $\alpha_k = (-1)^k\pi/3$ ($k = 1, 2$); 4) reinforcement in the asymptotic directions [7], which is typical of the SSOH:

$$\tan\alpha_k = (-1)^k\sqrt{RR''/(1 + (R')^2)}, \qquad k = 1, 2 \tag{3.10}$$

(in this case, the reinforcement trajectories are rectilinear).

Figures 1–3 show the distribution of the temperature T in the conical shell, the SSEP ($c = -0.01R^0$), and the SSOH ($c = 0.5R^0$), respectively, for various reinforcement variants and heat-exchange regimes (the dimensionless x is laid off as the abscissa). The temperature

$$T(x_1^0) = T(x_1^1) = 250°\text{C}, \tag{3.11}$$

is specified at the edge x_1^0 where the oil flows into the shell and $T_1(x_1^0) = 20$°C. Figure 4 shows the distribution of the fluid temperature T_1 in the conical shell (curves with the same numbers in Figs. 1–4 refer to the same reinforcement variant or heat-exchange regime). A similar distribution of T_1 is observed for the fluid that fills the tubes in the SSEP and the SSOH.

Curves 1 refer to the case where the heat exchange between the tubes and the fluid is absent (the internal surfaces of the tubes are thermally insulated or the tubes contain no fluid). In this case, the temperatures T and T_1 remain constant. Curves 2–4 characterize the temperature distribution in the oil and in the shells reinforced at different angles [$\alpha_k = 0$, $\alpha_k = \pm\pi/4$, and $\alpha_k = \pm\pi/3$ ($k = 1, 2$), respectively] for the same heat-exchange regimes ($\varepsilon = 2.7 \cdot 10^{-4}$, $V_1 = 40$, and $H_1 = 4$). For the same heat-exchange parameters, an increase in the reinforcement angle from 0 to $\pi/3$ leads to an abrupt decrease in the temperature of the shells (curves 2–4 in Figs. 1–3), whereas the fluid temperature remains almost the same (curves 2–4 in Fig. 4). The reasons are the following. As a criterion of comparison between the reinforcement variants, we chose the condition of equal total cross-sectional areas and the condition of equal internal cross sections of the tubes, i.e., the equality of the quantities ω_{*k} and Ω_{*k} (3.6) in all the variants. As the reinforcement angle increases

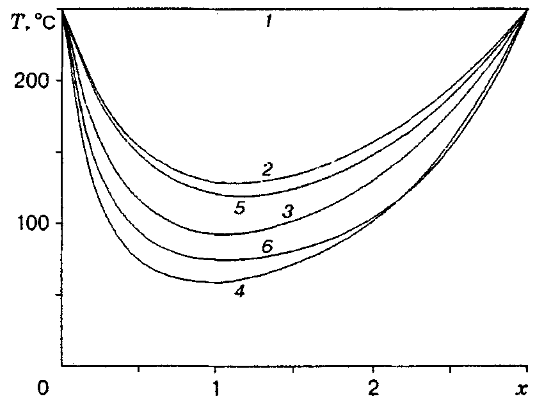

Fig. 1

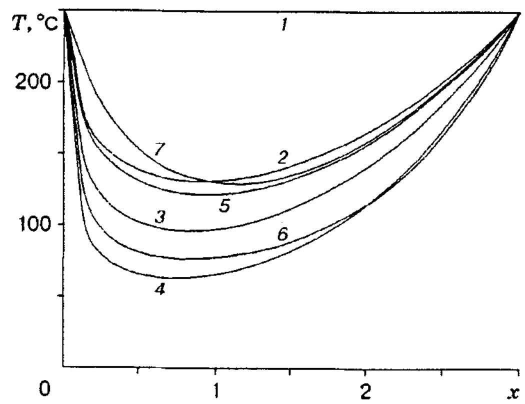

Fig. 2

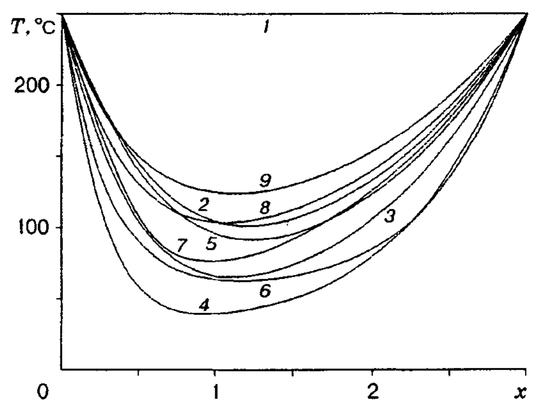

Fig. 3

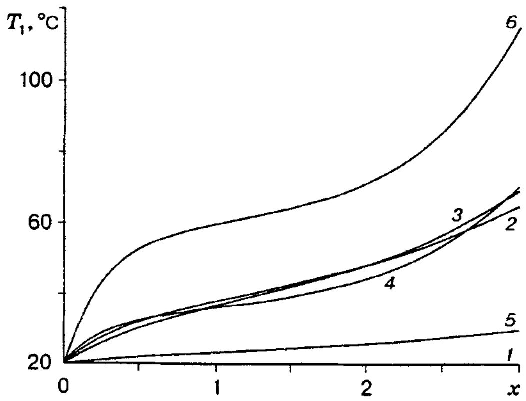

Fig. 4

from 0 to $\pi/3$, the length of the capillary tubes in the shell increases; provided the fluid flow rate remains the same, this leads to an increase in the duration of the heat exchange between the fluid element, which moves along the tube, and the tube wall. At the same time, the amount of shell material decreases (A decreases because of the increase in ω_k and Ω_k), which intensifies its cooling (curves 2–4 in Figs. 1–4).

Curves 5 and 6 characterize the temperature distribution in the shells reinforced in the meridional direction ($\alpha_k = 0$) but for other heat-exchange regimes. Curves 5 refer to the case where the oil flow rate is increased by a factor of 5 ($V_1 = 200$ and $H_1 = 4$). This leads to a slower increase in the fluid temperature T_1 [it follows from (3.8) that $T_1' \to 0$ as $V_1 \to \infty$ and $\varepsilon = 0$] and to a more intense heat exchange between the fluid and the tubes compared to the case represented by curves 2 in Figs. 1–4 ($\alpha_k = 0$, $V_1 = 40$, and $H_1 = 4$). Curves 6 correspond to the case where the internal diameters of the tubes are decreased by a factor of 5 ($V_1 = 40$ and $H_1 = 20$) for the initial fluid flow rate, whereas their number is increased by a factor of 25. In this case, the total area of the internal cross sections of the tubes remains the same (the functions ω_k and Ω_k are the same in all the structures reinforced in the meridional directions). As one would expect, the decrease in the cross-sectional area of the tubes and the increase in their number intensify the heat exchange between the shell and the fluid. Moreover, the outlet temperature of the oil in the conical shell increases by a factor of 2.12 compared to the case represented by curve 2 in Fig. 4. In all the shells considered, the minimum temperature almost halves (curves 2 and 6 in Figs. 1–3).

Curve 7 in Fig. 2 refers to the meridional reinforcement of the SSEP with a different geometry [$c = -10R^0$ in (3.2)] for the same heat-exchange regime ($V_1 = 40$ and $H_1 = 4$); curves 7–9 in Fig. 3 characterize the temperature distribution in the SSOH of different geometry [in (3.5), $c/R^0 = 0.5, 0$, and -1, respectively]

TABLE 1

Curve No.	Minimum temperature in the shell, °C		
	conical shell	SSEP	SSOH
1	250.0	250.0	250.0
2	128.6	130.4	100.9
3	92.6	96.1	65.2
4	59.0	62.6	38.6
5	119.6	121.3	91.3
6	74.4	76.5	62.3
7	—	129.2	76.2
8	—	—	103.3
9	—	—	123.8

reinforced in the asymptotic directions (3.10) for $V_1 = 40$ and $H_1 = 4$. Comparison of curves 2–4 and 7 in Fig. 3 shows that, for the same heat-exchange regimes, the reinforcement in the rectilinear asymptotic directions, which is the simplest from the practical viewpoint, does not ensure the most intense heat removal from the shell [in the sense that the temperature in the SSOH with spiral reinforcement at the angles $\alpha_k = \pm\pi/4$ and $\alpha_k = \pm\pi/3$ is smaller than that upon reinforcement in the asymptotic directions (3.10) (curves 3 and 4 in Fig. 3 lie below curve 7)].

However, not only the reinforcement structure but also the shape of the shell affects the temperature distribution. Table 1 lists the minimum temperatures for the conical shell, the SSEP, and the SSOH. Inspection of Table 1 shows that of the three types of shell, the SSOH is cooled most intensely and the SSEP is cooled least intensely for the same reinforcement structures and heat-exchange regimes. This is due to the following facts. Since the values of ω_{*k} and Ω_{*k} in (3.6) are the same in all the variants, the functions ω_k and Ω_k depend strongly on the radius of the shell $R(x_1)$ for the same angles of reinforcement α_k: the smaller the radius R, the greater the values of ω_k and Ω_k, and, hence, the lower the intensity of distribution of the basic shell material A [see (1.4)] and the more intensely it is cooled with other conditions being equal (the reinforcement structure and the heat-conduction regime). For example, as x_1 increases, the radius $R(x_1)$ of the SSOH with $c = 0.5R^0$ (curves 2–7 in Fig. 3) first decreases from the value of R^0 to the value of $R(c)$ (the parameter c in (3.4) determines the position of the throat line of the SSOH middle surface [7]) and then increases up to the value of R^1, but the values of the radius $R(x_1)$ do not exceed R^0 in the interval $x_1^0 \leqslant x_1 \leqslant 2c - x_1^0 = R^0$. The SSOH is cooled more intensely than the conical shell and the SSEP, since $R(x_1) > R^0$ for $x_1 > x_1^0$ for these shells. Moreover, the radius R of the conical shell varies as x_1 $[R'(x_1) = \text{const}]$, whereas the radius R of the SSEP with $c = -0.01R^0$ increases abruptly in the neighborhood of the edge x_1^0 $[R'(x_1^0) \overset{c\to -0}{\longrightarrow} +\infty]$, and at the points remote from this edge, R changes insignificantly and remains greater than the radius of the conical shell. Therefore, with other conditions being equal, the SSEP is cooled less intensely than the conical shell.

The shape of the generatrix affects both the magnitude and distribution of the temperature field in the shell. For example, in the SSEP (see Fig. 2) the minimum temperature points are located near the edge x_1^0, and in its neighborhood, the temperature gradient is greater than that in the conical shell (see Fig. 1) and the SSOH (see Fig. 3). Moreover, the minimum temperatures that correspond to curves 3 and 6 in Fig. 3 differ by 2.9°C and those in Figs. 1 and 2 differ by 18.2 and 19.6°C, respectively. Consequently, for certain values of the parameter c close to the limiting values ($c \to c^1 = 0.75R^0$), a portion of curve 3 lies below curve 6 for the SSOH (in Figs. 1–3, curve 6 lies below curve 3). Consequently, the choice of the reinforcement variant that ensures the most intense heat removal from the structure depends on the shell geometry.

As was noted above, the SSEP and the SSOH degenerate into conical shells as $c \to -\infty$ and $c \to c^0$, respectively [see (3.2)–(3.5)]. Indeed, curve 7 in Fig. 2 refers to $c = -10R^0$ and the meridional reinforcement; a comparison of this curve and curve 2 in Fig. 1 (the conical shell reinforced in the meridional direction) shows that the temperatures differ insignificantly, the difference between their minimum values being 0.6°C.

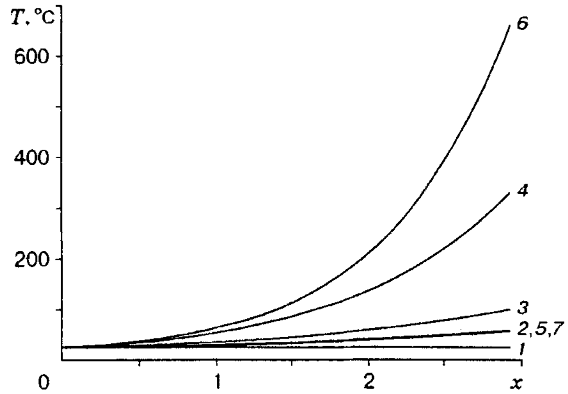

Fig. 5

Curves 8 and 9 in Fig. 3 refer, respectively, to $c/R^0 = 0$ and -1 of the SSOH reinforced in the asymptotic directions (as $c \to c^0 = -1.5R^0$, the SSOH degenerates into a conical shell and the asymptotic directions tend to meridional directions: $\alpha_k \to 0$). The minimum temperature values in curves 8 and 9 (see Fig. 3) differ from those in curve 2 (see Fig. 1) by -25.3 and -4.8°C, respectively. Consequently, as c tends to the limiting values [see (3.2)–(3.5)], the SSEP temperature is the upper bound and the SSOH temperature is the lower temperature bound in the conical shell for the corresponding reinforcement structures.

If the minimum condition for the lowest temperature is used as a thermophysical criterion of effective reinforcement of a tubular shell of fixed geometry, the spiral reinforcement with a winding angle $\alpha_k = \pm\pi/3$ is the best variant, which is seen from Figs. 1–3 (curves 4); if the shell geometry is varied, the best variant is the SSOH with the above-mentioned reinforcement, for which the minimum temperature is close to room temperature ($T_{\min} = 38.6$°C).

It is noteworthy that not only the reinforcement structure and the geometry but also the boundary conditions for the temperature affect the temperature distribution. We consider, for example, the SSEP with the initial geometrical parameters ($c = -0.01R^0$) and specify the temperature and the zero heat flux at the edge x_1^0:

$$T(x_1^0) = 25°\text{C}, \qquad q_1(x_1^0) = 0. \tag{3.12}$$

Figure 5 shows the temperature distribution in this shell for different reinforcement structures (the curve numbers correspond to those in Fig. 2). Comparing diagrams in Figs. 2 and 5, we infer that the change in the boundary conditions leads to quantitative and qualitative changes in the temperature field in the shell. For example, for boundary conditions (3.11) (see Fig. 2), the most intense heat removal from the shell corresponds to the spiral reinforcement with angles of $\alpha_k = \pm\pi/3$ (curve 4 in Fig. 2), whereas, for boundary conditions (3.12), this result corresponds to the meridional reinforcement ($\alpha_k = 0$) characterized by a large value of H_1 (curve 6 in Fig. 5). Consequently, the reinforcement structure that ensures the most intense heat removal from the shell for one set of boundary conditions might be not optimal for another set of boundary conditions. For boundary conditions (3.12) and different initial data, the discrepancy between the temperatures amounts to hundreds of degrees (see Fig. 5) instead of tens of degrees as in the case of boundary conditions (3.11) (see Fig. 2). Moreover, the effect of the shell shape is more pronounced for boundary conditions (3.12) than for boundary conditions (3.11). For example, for boundary conditions (3.12) and different initial data (V_1, H_1), the SSOH and SSEP temperatures differ by severalfold (by hundreds and thousands of degrees) instead of tens of degrees, which is the case of conditions (3.11) (see Figs. 2 and 3), the reinforcement structures and heat-exchange regime being the same.

Diagrams in Fig. 5 show that there exist wide possibilities of controlling the temperature field in tubular shells; therefore, various problems of target-oriented control can be formulated on a set of solutions of the heat-conduction problem. For example, if the temperature $T(x_1^1)$ and the heat flux $q_1(x_1^1)$ are specified at the edge x_1^1, one can obtain the required values of $T(x_1^0)$ and $q_1(x_1^0)$, say, $T(x_1^0)$ is room temperature and $q_1(x_1^0) = 0$ at the edge x_1^0, by varying the reinforcement structure. Two additional boundary conditions at the edge x_1^0 can be satisfied owing to the fact that the heat-conduction problem is linear and two free parameters of reinforcement exist: the direction α_k and the cross-sectional dimension of the tubes r_1 (or H_1). In the classical heat-conduction problems of solids, these conditions at the edges of a shell cannot be satisfied.

In summary, the temperature field in tubular composite structures like shells of revolution depends qualitatively and quantitatively on the reinforcement structure (α_k, ω_k, r_k), the thermophysical characteristics of the composition phases λ_c, λ_{ik}, c_k, and ρ_k (i, $k = 1$, 2), the fluid flow rate v_k, the geometry of the shell $R(x_1)$, and the thermal boundary conditions. This offers wide possibilities in designing effective reinforcement projects and structures; in so doing, it is necessary to formulate separate problems of target-oriented control of the reinforcement structures for shells of different geometry and different heat actions.

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